

Fluctuations of a one-dimensional directed polymer in a random potential with finite correlation radius

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1998 J. Phys. A: Math. Gen. 31 2591

(<http://iopscience.iop.org/0305-4470/31/11/009>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.121

The article was downloaded on 02/06/2010 at 06:28

Please note that [terms and conditions apply](#).

Fluctuations of a one-dimensional directed polymer in a random potential with finite correlation radius

S E Korshunov and Vik S Dotsenko

L D Landau Institute for Theoretical Physics, Russian Academy of Sciences, Kosygina 2, 117940 Moscow, Russia

Received 14 August 1997, in final form 21 November 1997

Abstract. We consider a one-dimensional directed polymer in a random potential which is characterized by Gaussian statistics with finite radius local correlations. It is shown that the well known Kardar's solution obtained originally for a directed polymer with δ -correlated random potential can be applied for the description of the present system only in the high-temperature limit. For the low-temperature limit we have obtained a new solution which is described by the one-step replica symmetry breaking. For the mean square deviation of the directed polymer of linear size L it provides the usual scaling $x^2 \sim aL^{2\zeta}$ with the wandering exponent $\zeta = 2/3$ and the temperature-independent prefactor.

1. Introduction

In a wide variety of physical systems one is interested in the behaviour of a fluctuating linear object (with finite line tension) interacting with a quenched random potential. The object under consideration may be a dislocation in a crystal, a domain wall in a two-dimensional magnet, a vortex line in a superconductor, a fluxon line in an extended Josephson junction and so on, but following [1] this class of problems is traditionally discussed in terms of a directed polymer in random media or simply as a 'directed polymer'.

Quite naturally the best understanding has been achieved for the simplest one-dimensional case when the displacements of a directed polymer can only occur in one direction. In such case a directed polymer in the continuous approximation can be described by the Hamiltonian

$$H[x(t), v] = \int_0^L dt \left\{ \frac{J}{2} \left(\frac{dx}{dt} \right)^2 + v[x(t), t] \right\} \quad (1.1)$$

where J is the linear tension, t is the longitudinal coordinate ($0 \leq t \leq L$) and $x(t)$ is the transverse displacement of a polymer with respect to a straight line. The simplest (or rather the most easily treatable) assumption on the distribution of the random potential $v(x, t)$ consists in taking it to be Gaussian with

$$\overline{v(x, t)} = 0 \quad \overline{v(x, t)v(x', t')} = 2V(x - x')\delta(t - t'). \quad (1.2)$$

Here and throughout an overbar denotes the average over the realizations of the quenched random potential.

Let us assume that at $t = 0$ the position of a polymer is fixed; $x(0) \equiv 0$. Then, the quantity of interest is the typical deviation of the polymer 'trajectory' $x(t)$ from the origin. More precisely, one wants to know the dependence on L of the average square deviation

of the polymer at the ending point $x = L$, which in the limit $L \rightarrow \infty$ is expected to be described by the simple scaling

$$\overline{\langle x^2(L) \rangle} \sim aL^{2\zeta} \quad (1.3)$$

where angular brackets denote the average over thermal fluctuations, and ζ is the so-called wandering exponent.

In the absence of the random potential the situation is trivial and the wandering exponent ζ is equal to $1/2$. In this case the trajectory deviates from the origin only due to the thermal fluctuations and the prefactor a in the scaling law (1.3) is proportional to the temperature, so at zero temperature $\langle x^2(L) \rangle = 0$.

In the presence of a quenched random potential the situation becomes much more complicated. Now besides the thermal fluctuations, the trajectory is pushed away from the origin also due to the randomness in the background potential landscape, so that the scaling law (1.3) may be governed by a new non-trivial wandering exponent. Moreover, since for a generic random potential the ground-state trajectory of the Hamiltonian (1.1) typically drifts away from the origin, the scaling law (1.3) can be expected to hold in the zero-temperature limit as well.

It is widely believed that for a whole class of *locally correlated* random potentials (such that the function $V(x)$ in (1.2) is quickly decaying for $|x| \rightarrow \infty$) the wandering exponent ζ is universal and equal to $2/3$. This conclusion is based on the exact results for finite temperature of [2, 3] and has also been confirmed by zero-temperature numerical simulations of the discrete version of the directed polymer problem [1, 4] (see also [5] for later references). However, both the calculation based on the reduction to the damped Burgers' equation with conservative random force [2] and the Bethe ansatz calculation in terms of replica representation [3] are only valid for the case of strictly δ -functional correlations of the random potential.

On the other hand, Mézard and Parisi have introduced a rather general self-consistent approach [6] which allows one to describe the fluctuations of a D -dimensional elastic manifold embedded in $(D + N)$ -dimensional random media. Application of this approach to the one-dimensional directed polymer ($D = 1$, $N = 1$) with Gaussian behaviour of the random potential correlation function [7] produces $\zeta = 3/5$, which contradicts the results of [1–3]. The other contradiction is that the solution considered in [5, 6] is characterized by broken replica symmetry, whereas the solution of [3] is replica symmetric. Therefore, a question can be posed whether the breaking of replica symmetry in the framework of the self-consistent approach is not an artefact imposed by the Gaussian form of a trial Hamiltonian.

In order to achieve a better understanding of the problem we present here our attempt to generalize the solution suggested by Kardar [3] for δ -functional correlations of random potential to a more physical situation when correlations of the random potential have finite correlation radius (in transverse direction).

In section 2 we review some details of the approach developed in [3] and use it to find the temperature dependence of the prefactor in equation (1.3) which turns out to be of the form

$$a \propto T^{-2/3}. \quad (1.4)$$

In section 3 the applicability of Kardar's solution [3] for the approximate description of the system with the finite radius r of the random potential correlations is discussed. We demonstrate that it can be used only in the high-temperature limit $T \gg T_0 \propto r^{2/3}$, whereas at low temperatures the solution has to have an essentially different structure which in principle could lead to a change in the wandering exponent. The impossibility to apply

the description with the help of Kardar's solution at arbitrarily low temperatures follows already from equation (1.4). One can expect that if in the low-temperature limit the typical trajectory goes away from the origin its drift should be determined by the quenched random potential and not by the effects of the thermal fluctuations. This means that in the system with reasonable short-scale *regularization* the divergence of the prefactor suggested by equation (1.4) at low temperatures can be expected to saturate.

In section 4 the low-temperature solution of the regularized problem is found for the particular choice of the random potential correlation function $V(x)$. The form of this solution can be described in terms of the effective one-step replica symmetry breaking ansatz. In this case one recovers the scaling law (1.3) with the same wandering exponent $\zeta = 2/3$ but with a *temperature-independent* prefactor.

2. The solution of the unregularized problem

2.1. The relation between exponents

The idea of Kardar's approach [3] is based on the indirect calculation of the wandering exponent ζ by analysing the scaling of the typical sample to sample *fluctuations* of the free energy. Suppose that the typical fluctuations of the free energy (produced by the random potential) scale is

$$\delta F \propto L^\omega \quad (2.1)$$

where the exponent ω is known. On the other hand, if the typical deviation of the trajectory from the origin is equal to x , then the loss of energy due to the elastic term in the Hamiltonian (1.1) must be of the order of Jx^2/L . Balancing the two energies, one can write the following estimate

$$\overline{\langle x^2 \rangle} \sim \frac{L\delta F}{J} \propto L^{\omega+1}. \quad (2.2)$$

Then, according to the definition of the wandering exponent (1.3), one finds the following simple relation between the two exponents

$$2\zeta = \omega + 1 \quad (2.3)$$

2.2. The replica method

The scaling of the free energy fluctuations with the size of the system L can be relatively easily investigated in terms of the replica method [8, 9]. To this end one has to calculate the average

$$Z(n) \equiv \overline{Z^n[v]} \quad (2.4)$$

of the n th power of the partition function

$$Z[v] = \int_{0 < t < L} Dx(t) \exp \left\{ -\frac{H[x(t), v]}{T} \right\} \quad (2.5)$$

obtained by integration over all the trajectories with $x(0) = 0$.

According to the definition of the free energy

$$F = -T \ln Z[v] \quad (2.6)$$

the replica partition function $Z(n)$ can be represented as follows

$$Z(n) = \exp \left[-\frac{n}{T} F \right]. \quad (2.7)$$

The average in equation (2.7) as elsewhere above is calculated over the realizations of random potential $v(x, t)$. On the other hand, the free energy $F \equiv F[v]$ is itself a sample-dependent random quantity, whose distribution function we denote as $P(F)$. Then (2.7) can be rewritten as

$$Z(n) = \int dF P(F) \exp \left[-\frac{n}{T} F \right] \quad (2.8)$$

which is the (generalized) Laplace transform of the free energy distribution function $P(F)$.

The free energy corresponding to the replica partition function (2.4) can be naturally defined as

$$F(n) = -T \ln Z(n). \quad (2.9)$$

Although this quantity can be calculated only for integer n , according to the standard ideology of the replica approach it has to be considered as a function of the *continuous* parameter n which implies a necessity of analytic continuation in n .

Let us represent the free energy $F(n)$ of the replicated system as a power series of the replica parameter n

$$F(n) = \sum_{p=1}^{\infty} \frac{F_p}{p!} n^p. \quad (2.10)$$

Then, taking the p th derivative over n at $n = 0$ from both sides of (2.9) for the p th order of the free energy fluctuations one finds

$$F_p = T^{1-p} \overline{\overline{F^p}} \quad (2.11)$$

where a double overbar denotes the irreducible average over disorder.

2.3. The Bethe ansatz type solution

In the framework of the replica approach the statistical mechanics of the *irregular* system is analysed by considering the statistical mechanics of the *regular* system in which the disorder manifests itself in the form of the interaction between n identical replicas of the original system. For the system described by the Hamiltonian (1.1) and the Gaussian statistics of the random potential (1.2), the averaging of $Z^n[v]$ over disorder leads to the expression for $Z(n)$, the form of which corresponds to the following replica Hamiltonian

$$H_{\text{repl}} = \int_0^L dt \left\{ \frac{J}{2T} \sum_{a=1}^n \left(\frac{dx_a}{dt} \right)^2 - \frac{1}{T^2} \sum_{a,b=1}^n V[x_a(t) - x_b(t)] \right\}. \quad (2.12)$$

Equation (2.12) has a form of the Euclidean (imaginary time) action describing the quantum-mechanical system of n particles with mass J/T and interaction $-V(x)/T^2$. The same system can be described by the quantum-mechanical (operator) Hamiltonian

$$\hat{H} = -\frac{T}{2J} \sum_{a=1}^n \nabla_a^2 - \frac{1}{T^2} \sum_{a,b=1}^n V(x_a - x_b) \quad (2.13)$$

which for the classical partition function defined by the Hamiltonian (2.12) plays the role of the transfer matrix.

In the limit of infinite size ($L \rightarrow \infty$), the free energy of a system (for any boundary conditions) is dominated by the highest eigenvalue of transfer matrix or, in our case, by the lowest eigenvalue E_0 of the quantum-mechanical Hamiltonian (2.13)

$$F(n) = T E_0(n) L. \quad (2.14)$$

For any integer n the lowest eigenvalue corresponds to the fully symmetric (nodeless) wavefunction which for the case of local correlations in the x direction

$$V(x) = u\delta(x) \tag{2.15}$$

has been found exactly by Kardar [3]

$$\Psi_0[x_a] = \exp\left(-\alpha \sum_{a,b=1}^n |x_a - x_b|\right) \tag{2.16}$$

where for our choice of notation

$$\alpha = \frac{Ju}{T^3}. \tag{2.17}$$

The energy of this state is equal to

$$E_0(n) = -\frac{V(0)}{T^2}n - \frac{Ju^2}{6T^5}n(n^2 - 1) \tag{2.18}$$

where the first term describes the trivial contribution to $E(n)$ related to the terms with $a = b$ in the second sum in Hamiltonian (2.13).

Substitution of (2.18) into (2.14) gives

$$F(n) = F_1n + \frac{1}{6}F_3n^3 \tag{2.19}$$

where

$$F_1 = \left[-\frac{V(0)}{T} + \frac{Ju^2}{6T^4}\right]L \tag{2.20}$$

and

$$F_3 = -\frac{Ju^2}{T^4}L. \tag{2.21}$$

Comparison of (2.11) with (2.19) shows that for $L \rightarrow \infty$ the average free energy (per unit length) of a random polymer

$$f \equiv \lim_{L \rightarrow \infty} \frac{\overline{F(L)}}{L} \tag{2.22}$$

is given by the linear in n contribution to $F(n)$

$$f = \frac{1}{L} \lim_{n \rightarrow 0} \frac{F(n)}{n} = \frac{F_1}{L} = -\frac{V(0)}{T} + \frac{Ju^2}{6T^4} \tag{2.23}$$

in which the first (formally divergent) term always dominates. Therefore, the average free energy of the system could be defined only after proper short-scale regularization of the original Hamiltonian.

However, the fluctuations of the free energy are quite well defined without any regularization. According to (2.11) and (2.21) the typical value of the free energy fluctuations can be estimated as

$$\delta F \sim \left(\overline{\left|\frac{F}{F^3}\right|}\right)^{1/3} = (T^2|F_3|)^{1/3} = \left(\frac{Ju^2}{T^2}\right)^{1/3} L^{1/3}. \tag{2.24}$$

Therefore, according to equation (2.2), for the average square deviation of the trajectory one finds the following result:

$$\overline{\langle x^2 \rangle} \sim \left(\frac{u}{JT}\right)^{2/3} L^{4/3}. \tag{2.25}$$

3. Introduction of the regularization

Apparently the divergence of the average free energy, equation (2.23), is removed if one takes into account that in a physical system correlations of random potential can be described by a smooth function with a finite correlation radius (and therefore a finite value of $V(0)$). However, in such a case the quantum-mechanical problem defined by equation (2.13) cannot be solved exactly. Nonetheless it seems reasonable to assume that for narrow enough $V(x)$ one can still use the expression (2.23) in which u now stands for

$$u = \int_{-\infty}^{+\infty} dx V(x). \quad (3.1)$$

It is easy to understand that such an approximate description (based on equations (2.16)–(2.21)) at low temperatures has to fail. One has to remember that in the case of the δ -functional interaction the wavefunction (2.16) is constructed as a generalization of a two-particle problem wavefunction

$$\Psi(x_1, x_2) = \exp(-2\alpha|x_1 - x_2|). \quad (3.2)$$

On the other hand, in the case of a rectangular well

$$V(x) = \begin{cases} V & \text{for } |x| < r \\ 0 & \text{for } |x| > r \end{cases} \quad (3.3)$$

(for which $u = 2Vr$) the wavefunction of the two-particle problem for $|x_1 - x_2| > r$ also has the form (3.2) with

$$\alpha = \frac{1}{r} g \left(\frac{2T_0^3}{T^3} \right) \quad (3.4)$$

where

$$g(z) \approx \begin{cases} z^{1/2} & \text{for } z \ll 1 \\ z & \text{for } z \gg 1 \end{cases} \quad (3.5)$$

and

$$T_0 = (JVr^2)^{1/3}. \quad (3.6)$$

It is not hard to check that the condition $\alpha r \ll 1$ for this wavefunction to be wide in comparison with the well width coincides with the condition $T \gg T_0$. In such a case the value of α given by equations (3.4)–(3.6) coincides with (2.17). In the opposite limit $T \ll T_0$ ($\alpha r \gg 1$) the two-particle wavefunction is almost completely localized inside the well and cannot be used as a building block for the construction of the solution of the n -particle problem.

This gives a clear indication that for the *arbitrary* finite-width form of the function $V(x)$, describing the correlations of random potential, the application of Kardar's solution for the description of random polymer can only work at high enough temperatures whereas in the low-temperature limit the solution is different.

Quite paradoxically differentiation of (2.23) shows that for $T \gg T_0$ the free energy defined by (2.23) corresponds to negative entropy. One should not be too scared of that property since in the approach discussed above the free energy of a directed polymer is calibrated in such a way that in the absence of the disorder it is equal to zero. Therefore, the total free energy will be given by (2.23) *plus* the free energy in the absence of disorder. This second term will give the positive contribution to the entropy which should overcome the negative contribution from (2.23).

4. The low-temperature solution of the regularized problem

Let us now consider the n -particle problem defined by the Hamiltonian (2.13) where

$$V(x) = V \left[1 - b \frac{x^2}{r^2} \right] \quad (4.1)$$

for $|x| < r$ and equal to zero elsewhere. Here, in comparison with (3.3) we have introduced a finite curvature of the potential inside the well, which is described by an additional free parameter b ($0 < b \leq 1$). At some stage of the calculation, b will be assumed to be much smaller than one. For $V(x)$ of the form (4.1)

$$u = 2 \left(1 - \frac{b}{3} \right) Vr \sim 2Vr \quad (4.2)$$

and, therefore, the characteristic temperature T_0 defining the range of applicability of Kardar's approach ($T \gg T_0$) can still be chosen in the form (3.6).

The characteristic frequency for small oscillations at the bottom of such truncated parabolic well is given by

$$\Omega = \sqrt{\frac{2bV}{JT r^2}} \quad (4.3)$$

which increases with a decrease of T much slower than the depth of the well $W = V/T^2$. Thus, the limit of low temperatures may correspond to the case when all particles are localized near the bottom of the well. In such a limit the ground-state energy $E(n)$ for the n -particle system can be rather accurately found by assuming that equation (4.1) holds for all $x_a - x_b$.

In such an approximation the ground-state wavefunction has a form

$$\Psi[x_a] = \exp \left[-\frac{1}{A\sqrt{2n}} \sum_{a,b=1}^n (x_a - x_b)^2 \right] \quad A = \frac{\Omega r^2}{W b} \quad (4.4)$$

whereas the energy of the ground state is given by

$$E(n) = -Wn^2 + \Omega \sqrt{\frac{n}{2}} (n-1) \quad (4.5)$$

(cf [9, 10]). In the following it will be convenient to keep in mind that the ratio of Ω and W can be expressed as

$$\frac{\Omega}{W} = \left(2b \frac{T^3}{T_0^3} \right)^{1/2}. \quad (4.6)$$

It is not hard to find by a straightforward calculation that for $\Psi(x)$ of the form (4.4)

$$\langle (x_a - x_b)^2 \rangle = \frac{A}{\sqrt{2n}} = \frac{1}{\sqrt{2n}} \frac{\Omega r^2}{W b} \quad (4.7)$$

so that for $\Omega/W \ll b$ (i.e. $T \ll b^{1/3} T_0$) $\Psi(x)$ is indeed nicely localized at the bottom of the well for any integer n and, therefore, all corrections to $E(n)$ due to non-parabolicity can only be exponentially small.

On the other hand, one can easily see why the limit $n \rightarrow 0$ is dangerous. The width (4.7) of the wavefunction (4.4) grows with a decrease in n and becomes comparable with the width of the well $2r$ at $n \sim b^{-1}(T/T_0)^3$ and therefore for smaller n the ground-state wavefunction should have an essentially different form. The simplest way to let the particles enjoy their mutual attraction while keeping their number in the well not too small consists

of splitting them into n/k infinitely separated blocks of k particles. The energy $E(n, k)$ of such a state with broken replica symmetry is given by

$$E(n, k) = \frac{n}{k} E(k) = n \left[-Wk - \Omega \frac{1-k}{\sqrt{2k}} \right] \quad (4.8)$$

and has extremum (maximum) as a function of k .

Variation of the polymer free energy per unit length

$$f(k) = T \left[-Wk - \Omega \frac{1-k}{\sqrt{2k}} \right] \quad (4.9)$$

with respect to k gives an equation for the position of the maximum

$$-W + \Omega \frac{1+k}{(2k)^{3/2}} = 0 \quad (4.10)$$

the solution of which for $\Omega/W \ll 1$ has the form

$$k_* \approx \frac{1}{2} \left(\frac{\Omega}{W} \right)^{2/3} = \frac{1}{2} (2b)^{1/3} \frac{T}{T_0}. \quad (4.11)$$

Substitution of equation (4.11) into (4.7) then shows that for

$$T \ll b^{2/3} T_0 \quad (4.12)$$

the replicas belonging to the same block are indeed tightly bound to each other:

$$\langle (x_a - x_b)^2 \rangle \approx \left(\frac{\Omega}{W} \right)^{2/3} \frac{r^2}{b} \approx \frac{T}{b^{2/3} T_0} r^2 \ll r^2 \quad (4.13)$$

so the whole picture is really self-consistent.

Substitution of equation (4.11) into (4.9) gives the temperature-independent expression

$$f \approx -\frac{3}{2} (2b)^{1/3} \frac{V}{T_0} \quad (4.14)$$

which shows that in order to find the temperature dependence of f in low-temperature limit we have to solve (4.10) more accurately. This gives

$$k_* \approx \frac{1}{2} \left(\frac{\Omega}{W} \right)^{2/3} + \frac{1}{4} \left(\frac{\Omega}{W} \right)^{4/3} \quad (4.15)$$

and

$$f \approx -\frac{3}{2} (2b)^{1/3} \frac{V}{T_0} + \frac{1}{4} (2b)^{2/3} \frac{V}{T_0^2} T. \quad (4.16)$$

The idea to consider the state in which n replicas are split into n/k infinitely separated blocks of k particles was introduced by Parisi [12], who, however, applied it only to the case of local interaction (2.15) (δ -functional correlations in terms of the original problem) and discovered that the free energy as a function of k has extremum at $k = 0$. This is equivalent to considering all replicas belonging to the same block from the beginning. Our analysis shows that smearing of the interaction potential leads (at low enough temperatures) to a shift of the extremum to non-trivial values of k ($0 < k < 1$) corresponding to splitting of the replicas into blocks (i.e. to replica symmetry breaking).

Although we have found that in the extremal solution the particles split into n/k separate blocks there is no reason for these blocks to be infinitely separated from each other. The presence of a strong attraction between the particles in each block makes it possible to consider each block as a complex particle with mass kJ/T , the interaction between these

complex particles being given by $-k^2V(x)/T^2$. The last expression can be expected to be very accurate when we consider the temperature interval (4.12) in which the distances between the particles inside each block are much smaller than the well radius r .

Therefore, at low temperatures the behaviour of our system in which the particles are assumed to be tightly bound in n/k separate blocks can be described by the Hamiltonian (2.13) in which

$$J \rightarrow kJ \quad V(x) \rightarrow k^2V(x) \quad u \rightarrow k^2u. \tag{4.17}$$

Our earlier experience tells us that for some values of parameters such a system can be rather accurately described by the wavefunction of the form (2.16) in which x_a now stands for the coordinates of different blocks. The energy of such a state will be given by equation (2.18) in which substitutions (4.17) and $n \rightarrow n/k$ have to be made with the first term being substituted by (4.8)

$$E(n, k) = n \left\{ -Wk - \Omega \frac{1-k}{\sqrt{2k}} - 2B \frac{W^3}{\Omega^2} k^4 \left[\left(\frac{n}{k} \right)^2 - 1 \right] \right\} \tag{4.18}$$

where

$$B = \frac{2b}{3} \left(1 - \frac{b}{3} \right)^2 \tag{4.19}$$

is a small parameter if b is small.

All this leads to the appearance in the expression for $f(k)$ of one more term (in comparison with equation (4.9))

$$f(k) = T \left[-Wk - \Omega \frac{1-k}{\sqrt{2k}} + 2B \frac{W^3}{\Omega^2} k^4 \right] \tag{4.20}$$

which describes the contribution related to the mutual interaction between the blocks. Substitution of (4.11) into the saddle-point equation

$$-W + \Omega \frac{1+k}{(2k)^{3/2}} + 8B \frac{W^3}{\Omega^2} k^3 = 0 \tag{4.21}$$

which is obtained by a variation of (4.20) shows that for $B \ll 1$ (that is for $b \ll 1$) the maximum of $f(k)$ still exists and in the lowest order in b the position of this maximum is still given by (4.11).

The applicability of such an approach requires that the distances between the blocks should be much larger than the size of the well (in exactly the same way as when a Kardar-type solution is constructed from the separate particles and not from the blocks), i.e.

$$\varkappa r \ll 1. \tag{4.22}$$

Substitution of equations (2.17), (4.17) and (4.11) into (4.22) then reduces it to the condition

$$\frac{b}{2} \left(1 - \frac{b}{3} \right) \ll 1 \tag{4.23}$$

which apparently is equivalent to the same condition $b \ll 1$.

Strictly speaking the expression for $f(k)$ given by (4.20) also has another extremum (minimum) at

$$k = k_{**} \approx \frac{1}{B^{1/3}} k_* \tag{4.24}$$

but analogous analysis shows that

$$\varkappa(k_{**})r \approx \frac{3}{4} \left(1 - \frac{b}{3}\right)^{-1} \sim 1 \quad (4.25)$$

and therefore this second extremum takes place in the domain of the parameters where expression (4.20) based on the assumption (4.22) can no longer be trusted.

The form of equation (4.18) shows that in the considered case the only nonlinear (in n) contribution to $E(n)$ is also of third-order in n and corresponds to

$$F_3 = -\frac{4VT_0^3}{T^4}k^2L \quad (4.26)$$

where we assume $b \ll 1$. Using the saddle-point value of the parameter k one gets

$$F_3(k_*) = -(2b)^{2/3} \frac{VT_0}{T^2}L. \quad (4.27)$$

Correspondingly, for the typical value of the free energy fluctuations one obtains the following temperature-independent result

$$\delta F \sim (T^2|F_3|)^{1/3}L^{1/3} = C_0L^{1/3} \quad (4.28)$$

where

$$C_0 = (2b)^{2/9}(VT_0)^{1/3}. \quad (4.29)$$

Finally, for the mean square deviation of the polymer trajectory one again finds $\overline{\langle x^2 \rangle} \sim aL^{4/3}$ with the temperature-independent prefactor

$$a = \frac{C_0}{J} = \left(2b \frac{V^2r}{J^4}\right)^{2/9}. \quad (4.30)$$

5. Conclusion

Thus we have demonstrated that in the case when random potential correlations are characterized by the finite correlation radius r the solution at low temperatures has an essentially different structure to that at high temperatures. Nonetheless, the value of the wandering exponent in both cases is the same; $\zeta = 2/3$. In contrast to the high-temperature limit for which the prefactor a in the scaling law (1.3) is temperature-dependent, $a \propto T^{-2/3}$, in the low-temperature limit it saturates at finite value $a \propto r^{2/9}$.

Note, however, that for $b \ll 1$ the value of a achieved at $T \ll b^{2/3}T_0$ (see (4.30)) turns out to be much smaller than its value at $T \sim T_0$ which can be estimated by substituting equation (3.6) into (2.25). Therefore, in the intermediate temperature region $b^{2/3}T_0 \ll T \ll T_0$ the prefactor a has to *decrease* with a decrease in temperature. Comparing the two estimates it is not hard to conclude that if the temperature dependence of a in this region is algebraic then it has to be of the form $a \propto T^{1/3}$.

Since the value of the wandering exponent ζ is the same both in the high-temperature and in the low-temperature limits there are no reasons for the sharp transition between different regimes. At very low temperatures the solution is characterized by the one-step replica symmetry breaking, that is the replicas are split into well separated blocks. With growth of temperature the distance between the blocks becomes comparable with the size of each block. At high temperatures the replica symmetry breaking phenomena can manifest itself only in the slight modulation of the distance between nearest replicas in comparison with what follows from the ‘replica symmetric’ wavefunction (2.16).

The results of this work testify that the appearance of replica symmetry breaking in the framework of a self-consistent approach [5, 6] is not an artefact but an essential feature of the solution. However, since our solution is characterized by one-step replica symmetry breaking with essentially non-Gaussian correlations between the positions of the blocks, the inability of the self-consistent approach to reproduce the same value of wandering exponent ζ can probably be related to the restrictions imposed by the Gaussian form of a trial Hamiltonian. On the other hand, a too direct comparison of our results with that of [7] is probably inappropriate since in our terms the Gaussian form of $V(x)$ corresponds to $b \sim 1$ and does not allow for the construction of the low-temperature solution consisting of really well separated blocks.

To avoid confusion it will be worthwhile to emphasize that the solution suggested in this work does not allow for further generalization to the so-called hierarchical replica symmetry breaking [13]. For each pair of replicas we have to assume that the average distance between them is either much smaller than r (in the same block) or much larger than r (in different blocks) which in our opinion leaves no other possibilities than one-step replica symmetry breaking.

Acknowledgments

The authors are grateful to M Mézard for useful discussions. This work has been supported in part by the Russian Foundation for Basic Research (grant Nos 96-15-96920 (VSD) and 96-02-018985), the Russian Federation Ministry of Science (Programme 'Statistical Physics' (SEK)) and INTAS (SEK).

References

- [1] Kardar M and Zhang Y-C 1987 *Phys. Rev. Lett.* **58** 2087
- [2] Huse D A, Henley C L and Fisher D S 1985 *Phys. Rev. Lett.* **55** 2924
- [3] Kardar M 1987 *Nucl. Phys. B* **290** 582
- [4] Huse D A and Henley C L 1985 *Phys. Rev. Lett.* **54** 2708
- [5] Halpin-Healy T and Zhang Y-C 1995 *Phys. Rep.* **254** 215
- [6] Mézard M and Parisi G 1990 *J. Phys. A: Math. Gen.* **23** L1299
Mézard M and Parisi G 1991 *J. Physique I* **1** 809
- [7] Blum T 1994 *J. Phys. A: Math. Gen.* **27** 645
- [8] Emery V J 1975 *Phys. Rev. B* **11** 239
- [9] Edwards S F and Anderson P W 1975 *J. Physique F* **5** 239
- [10] Parisi G 1990 *Rend. Acad. Naz. Lincei* **XI-1** 277
- [11] Maritan A, cited by [5] unpublished
- [12] Parisi G 1990 *J. Physique* **51** 1595
- [13] Mézard M, Parisi G and Virasoro M A 1987 *Spin Glass Theory and Beyond* (Singapore: World Scientific)